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CAN A WELD IN WELDED STRUCTURE BE MADE WITH ZERO RESIDUAL STRESS?

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1 Abstract

While solving a sequence of seventeen optimization projects to predict the values of the side heater parameters that would be expected to minimize camber distortion in an edge welded bar, the design parameters that reduced distortion to effectively zero were not unique. This raised the question if any of the designs that minimized the distortion effectively to zero also minimized the residual stress. To answer this question three different measures of residual stress were evaluated for all 1451 designs. The Computational Weld Mechanics (CWM) optimization problem is to find the best point in the 4D space of side heater design parameters: flux, heated area, longitudinal and transverse distance from the weld such that the final residual stress is as low as possible (minimized). To evaluate the objective function for each point in the 4D design space, the associated 3D transient non-linear thermal visco-elastic- plastic stress analyzes was solved. A FEM mesh with 6600 8-node brick elements and 9438 nodes was solved for 166 time steps in 10 minutes of single-core CPU time. In the seventeen optimization projects, 1451 weld analyses were solved in 75 quad-core CPU hours by one person in two calendar weeks. The residual stress was effectively reduced to zero in some designs. These designs also reduced distortion to effectively zero. Whether a design that effectively reduces the residual stress to zero is unique remains an open question.

Key words: weld residual stress, welding distortion, side heaters, optimization, design of experiments (DOE), direct search, objective function, response surface.

2 Introduction

Section three provides background on the residual stress in welded structures. Section four briefly describes the optimization methodologies that are used. Section five describes the problem being analyzed. Results are presented in Section six. Section seven is the conclusion.

3 Background

The overview [1] of the current state of the art in fracture and fatigue in welded joints and structures makes it clear that residual stress in welds is an important factor in evaluating the risk of failure in the design of welded structures. Usually a weld has a longitudinal stress of yield stress magnitude after welding is complete and the structure has cooled to ambient temperature. This tensile residual stress can increase the risk of fatigue and stress corrosion cracking, creep failure and brittle fracture.

The most common way to reduce the risk of failure due to residual stress in a welded structure after welding has been completed is to stress relieve the welded structure by placing the structure in a furnace and heating it to a temperature at which residual stresses relax because the yield stress is reduced at the higher temperatures or the temperatures are high enough that stress relaxation or creep is fast enough in the time at temperature to reduce residual stress significantly. For low-alloy steel

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structures, the tempering temperature is slightly below the eutectoid temperature so that the austenite phase does not form. (This process also tempers any martensite that might be present in low-alloy steel welds and this usually increases the toughness of the material.) This furnace stress relief is expensive. In complex structures the heating and cooling process must manage thermal stress to avoid introducing new residual stress and distortion on cooling from the stress tempering temperature.

To deal with designs in which the welds cannot be tempered, the temper bead welding process has been developed. In this process, the welding procedure is managed so that the subsequent weld passes temper the microstructure and reduce the hardness of the previous weld passes. Temper bead welding is designed to manage weld hardness but is not designed to minimize residual stress [2].

It has become a common practice in pressurized water nuclear reactors to apply an overlay weld on the outside of a pipe or nozzle girth weld. This acts like a rubber band generating a compressive stress on the inside wall of the girth weld. This compressive stress reduces the risk of stress corrosion cracking associated with residual stress. Various forms of peening are another approach used to generate compressive stresses on the surface of a weld in an attempt to reduce the risk of fatigue cracking.

Many papers have been published about minimizing distortion in welded structures and many papers have been published on stress relief to reduce residual stress in welded structures. See [3] and the references therein for a recent review of the state of art. To the author's knowledge very few papers have been published on how to design welding procedures that generate zero or low residual stress in welds. An exception is the few papers on welding steel structures using a filler metal that transforms to a martensite at low enough temperature so that the volume expansion of the austenite to martensite transformation reduces the final residual stress state. See [4] [5] and references therein for details.

It is not clear why relatively few papers have been published on designing weld procedures to minimize residual stress. One possible reason for the lack of research in this area is that it is more difficult to measure residual stress than to measure distortion. In particular many techniques for measuring residual stress are point techniques, e.g., neutron and x-ray diffraction, strain gauges and various cutting techniques such as deep hole drilling, that measure some components of residual stress at a point. However, the contour method provides a measurement of the residual stress component normal to a cut-surface.

Another relevant issue is that design codes, such as the ASME Pressure Vessel code, are primarily based on a concept known as Design by Rule in which designers compute the stress in a structure using essentially handbook formulae. These formulae are idealized approximations based on best practices built up over decades. These stresses can be quite different from the stress that would be computed with an accurate 3D nonlinear transient finite element analysis. In recent decades, some codes have permitted the option of Design by Analysis. For this option designers must convince the Code Body that their analysis satisfies the code requirements.

4 Optimization and Optimal Design of Welded Structures

G. Strang said it best on page 666 in his book, Introduction to Applied Mathematics [6]: "This chapter also completes the last of the three fundamental areas of applied mathematics: **static problems, dynamic problems** and **optimization**. There is no doubt that optimization requires the most finesse. It leads to the *best* design, while the others analyze a *given* design. In statics and dynamics the materials and the shapes and equations were given; now we have to find the equations before solving them."

Designs can be categorized as parameterized or non-parameterized. For a parameterized design a set of design variables is specified with an upper bound, lower bound and initial trial value for each design variable. To define a particular design, the designer simply sets the value of each design parameter. This defines a design space, i.e., a space for all allowed variations of this design. Parameterizing a design space makes it easier to understand, learn, analyze, optimize, regulate and manage. Software can be developed for parametric designs that requires almost no training to use the software. As always designers should be experts in the things they are designing but they should not have to be experts in using any particular software package. Girth welds on pipelines and nozzles in pressure vessels are examples of designs that can be parameterized easily. If the design is not parameterized, more training is required to use the software. In particular, for welded structures with complex geometry, creating the FEM mesh for each variation of the design can be a challenge that requires a significant level of expertise. All designs analyzed in this paper were parameterized.

Optimization methods are often classified by whether they evaluate only the objective function, or the objective function and its first derivative with respect to state and design variables or the objective function, its first and second derivatives wrt to state and design variables. Because writing software to evaluate derivatives can be a difficult, time consuming task, it is rarely done for welding problems. See [7] for one of the few published examples. There is also an issue that the optimization problem might not be smooth enough for the derivatives to exist. For these reasons, we prefer to use optimization methods in which only the objective function is evaluated [25]. VrWeld has support to optimize any problem that is parameterized by parameters in our software for analyzing welds and welded structures, i.e., a reference project must be implemented that is contained in the design space, the design space is defined by entering the name of each design parameter and the lower and upper bound and smallest and largest incremental change in a design parameter, and an objective function is defined, usually by a short script in a scripting language. Finally, the user specifies the number of samples in the initial set of designs and the number of designs in each subsequent set of designs and the number of subsequent sets to be optimized. Using Kriging or local optimization, the program halts when it bas completed the analysis of all designs. Then post-processing allows the user to choose the best design or to explore the variations of the objective function in the design space.

4.1 Objective Functions

To optimize a welded structure, one or more objective functions, in effect goals, are specified that are to be minimized such as the cost or are to be maximized, such as the life of the welded structure. A computer model of the weld is generated for the initial trial or reference design. Then the software can evaluate essentially all variations of the design in that design space and choose the design in the space that minimizes the objective function. One can also choose the most robust design, i.e., the best design that is the least sensitive to changes in design variables over its service life. Such changes could be due to corrosion, wear, fatigue, etc.

Another difficulty with minimizing residual stress in welded structures is choosing the objective function that is to be minimized, i.e., a scalar function that can be evaluated for a given design or experiment. An obvious choice is the life of the structure but this requires a knowledge of service loads. In this paper, we consider the following three different objective functions. These are evaluated for each design after welding was completed and the structure had cooled to room temperature.

1. Maximum Effective Stress: For most metal, the yield function for plastic deformation satisfies a von Mises criteria or J_2 flow stress criteria. For rate independent plasticity this criteria constrains the deviatoric stress state to be less than or equal to the yield stress. The farther the maximum effective stress is from the boundary of the yield stress, the lower the risk of plastic deformation. Plastic deformation can be associated with the evolution of damage, e.g., the Coffin-Manson equation for low-cycle fatigue. The maximum effective stress can also be associated with shear band formation and associated failure mechanisms.
2. The maximum tensile stress or more precisely the maximum principal tensile stress: This criteria is often associated with increased risk of brittle fracture and cleavage.
3. The maximum tensile hydrostatic stress: This criteria is important in ductile fracture and creep. Higher tensile hydrostatic stress encourages the formation of voids. This increases the risk of ductile fracture and creep failure.

4.2 Direct search algorithm

Kolda, Lewis and Torczon have proposed a basic algorithm called "*Compass Search*" and an advanced one called "*GSS*" for direct search optimization in [8]. Their ideas are summarized in the pseudo-code below. An objective function, a DOE-matrix framework and a starting point is given. (A DOE-matrix framework is a set of design parameters with bounds on each parameter and a minimum step size and trial step size for each parameter).

1. Choose design variable values for an initial trial or reference design solution.
2. Create a template DOE matrix centred at the current best trial solution that respects the constraints. A Hyper-Cross template creates two additional rows in the DOE matrix for each design variable by one positive and one negative increment of the design variable for the current increment size. This template has $2N+1$ rows for N design variables. A Hyper-Cube Template has 3^N rows in the template.
3. For each row in the DOE matrix, solve the CWM problem defined by the row and evaluate the objective function.
4. If the objective function in one of the rows of the templates is a new minimum with a sufficient decrease:
   Move to the minimum as a new trial point and go to 2.
5. If there is not a new minimum point in the solutions for this template:
   Test for convergence.
   If no convergence, refine the increment step size and go to 2.
   If convergence, write report and stop.

VrWeld implements this algorithm to solve optimization problems. For each starting point, the algorithm steps toward the local optimum. If there is only one local optimum, then this algorithm is likely to find it if it takes sufficiently long steps. If there is more than one local optimum, then this algorithm can at most find the local optimum in the basin of attraction that contains the starting design point for this local optimization. The method implicitly uses a grid defined by the minimum step size. Therefore, the best it can do is find the closest point on the grid.

4.3 Uniform Discretizaton of a Design Space

A simple global optimization algorithm is to discretize the design space with a uniform grid. In 2D a 17×17 grid has 289 designs. The computational cost might be justified by the gain in simplicity. However, this becomes expensive in higher dimensional design spaces, e.g., in 4D a $17 \times 17 \times 17 \times 17$ grid has 89,521 points. This method is also restricted to values that lie on the grid.

4.4 Kriging

The mathematical theory of kriging was developed by Matheron in the 1960s based on Diane Krige's 1951 M.Sc. thesis. It can be viewed as a method of global optimization [17]. Given a design space and the values of the objective function an initial set of points in the design space, kriging provides the best linear unbiased prediction of intermediate values. Kriging can be viewed as a form of Bayesian inference. It provides a method to predict where the next point in the space should be measured to minimize the global uncertainty. It does not require a grid. Therefore it can, in principle find the values of the optimal design variables to higher precision than methods that are limited to design variables on a grid, i.e., to a finite set of values. A metamodel or surrogate model can be computed from the global optimization data points computed by kriging. Kriging is more efficient than most algorithms in higher dimensional design spaces.

5 The Welding Problem and Computational Weld Mechanics Analysis

The test setup is an edge weld on a 152.4 x 1220 x 12.7 mm bar of Aluminum 5052-H32. The full computational model that includes transient thermal and stress analysis is analyzed for each design point [18]. CWM validation for this edge weld is described in detail in [19] . The validation compared experimental data measured carefully by Masubuchi in [20] with data predicted by the computer model. Figure 1

The mesh employed, which is shown in Fig. 2, has 6600 8-node brick elements and 9438 nodes. The material was aluminum 5052-H32 alloy with chemical composition Al 96.7, Mg 2.5, Cr 0.25, Cu max 0.1, Fe max 0.4, Mn max 0.1, Si max 0.25, Zn max 0.1 Wt %. The temperature dependent material properties of Al 5052-H32 were given in [20] and this data was employed in the analysis of this test. The gas metal-arc-welding process parameters were current 260 A, voltage 23 V, travel speed 7.1 mm/s. The specimen was allowed to cool to ambient temperature $300^{\circ}K$ after welding was completed.

There are errors in the numerical analysis. Reference [19] describes the validation of the computer model for computing the camber in the edge weld of this bar with no side heater. shows that the model quite accurately predicted the transient mid-length deflection. This suggests that the errors in the model with no side heaters are relatively small.

VrWeld Inputs and Outputs

The input data to VrWeld are:

1. The geometry of the structure being welded, the geometry of the weld joint and filler metal added for each weld pass. The position of each weld pass, its starting point, arc speed and ending point.
2. Temperature dependent thermal and mechanical material properties of base metal and weld HAZ and fusion zones.
3. The values of the weld process parameters including weld current, weld voltage and filler metal wire size and speed are required. The start time of each weld pass was computed based on the start time of the first weld pass.
4. A cool-down period after each weld pass has been completed to allow the bar to cool to ambient temperature.

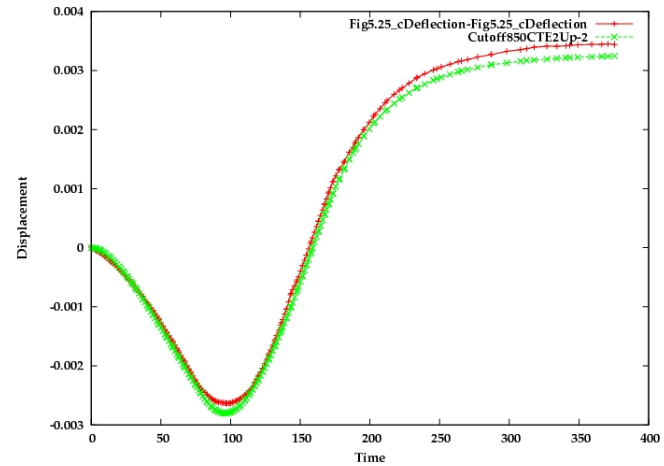

FIGURE 1: Comparison of the measured mid-length deflection and computed mid-length deflection with settings of cut-off temperature to 850 K, convergence criteria to 1e-6, maximum number of NR iterations to 10 and an increase in CTE by adding 2e-6 to the original CTE values taken from [20].

The output data from VrWeld include the following fields for all nodes (or Gauss points) in the mesh and all time steps:

1. Transient temperature field.
2. Transient displacement field.
3. Transient stress tensor field and principal stress fields as scalar components or vectors.
4. Transient elastic, thermal, plastic and total strain vector or tensor fields.

What Equations Does VrWeld Solve?

Conservation of Energy or Heat Equation With specific enthalpy h, thermal flux q and a power density function Q, temperature T, temperature gradient ∇T, thermal conductivity tensor κ specific heat c_p, the heat equation can be be written in the following form:

$$\begin{aligned} \dot{h} + \nabla \cdot q + Q &= 0 \\ q &= -\kappa \nabla T \\ dh &= c_p dT. \end{aligned}$$

VrWeld solves this partial differential equation on a domain defined by an FEM mesh. The domain is dynamic in that it changes

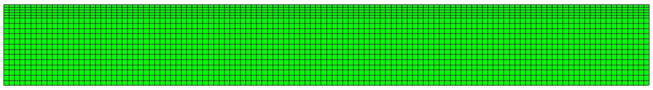

FIGURE 2: A 2D view of the 3D mesh employed in the analysis.

with each time step as filler metal is added to the weld pass. The initial condition is assumed to be a constant temperature of 300°K. The material properties κ and c_p are temperature and microstructure dependent. The heating effect of the arc is often modelled by a double ellipsoid power density distribution that approximates the weld pool as measured from macro-graphs of the cross-section of several weld passes [21]. A convection boundary condition $q = h(T - T_{amb})$ is applied to external surfaces. The FEM formulation of the heat equations leads to a set of ordinary differential equations that are integrated in time using a backward Euler integration scheme. More details on the methodology for computing transient temperature fields in welds can be found in [9] [19].

Conservation of Momentum Equation Given the density, ρ, the elasticity tensor as a 6×6 matrix, the body force b and the Green-Lagrange strain ε, VrWeld solves the conservation of momentum equation that can be written in the following form in which inertial forces, $\rho\ddot{x}$ are ignored.

$$\begin{aligned} \nabla \cdot \sigma + b &= 0 \\ \sigma &= D\varepsilon \\ \varepsilon &= (\nabla u + (\nabla u)^T + (\nabla u)^T \nabla u)/2 \end{aligned}$$

VrWeld solves this partial differential equation for a visco-thermo-elasto-plastic stress-strain relationship using theory and algorithms developed by J.C. Simo and his colleagues [23]. More detailed discussion of the thermal stress analysis of welds can be found in [9] and [24]. The initial state is usually assumed to be stress free. However, if the initial stress state was known, it could be initialized in VrWeld. The Dirichlet boundary conditions constrain the rigid body modes. The system is solved using a time marching scheme with time step lengths of approximately 1 second during welding and usually an exponentially increasing time step length begins when welding stops and applied until the structure cools to ambient temperature.

6 Results

The stress analysis for the weld without side heaters shows a significant Y-deflection as illustrated in Fig. 3. The goal of the first set of seventeen optimization projects was to mitigate this deflection in order to get as straight a bar as possible when welding is complete. The goal of this paper is to determine if any of the 1451 designs analyzed in the first set of seventeen optimization projects minimize the residual stress. Fig. 3 shows the transient temperature field when the weld is at the mid-point for the design with no side heaters, design A and design B with side heaters. Design A was chosen as the design of side heaters that minimized residual stress in the norms shown in Fig. 4, 5 and 6.

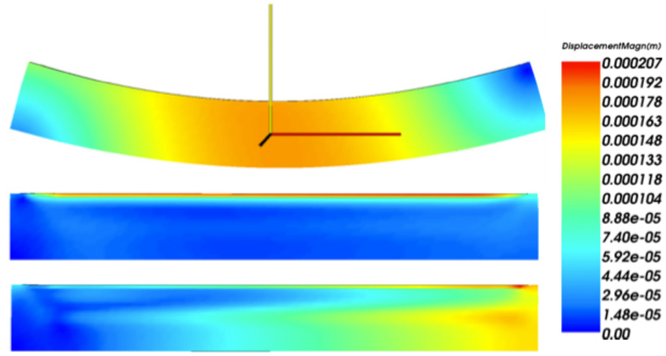

FIGURE 3: The top image shows the deflection (magnified x30) in Y direction at the end of the welding process with no side heaters. Red and yellow axes are X and Y. The middle image shows the deflection (magnified x30) with camber minimized by side heaters for Design A. The bottom image shows the deflection (magnified x30) with camber minimized by side heaters for Design B.

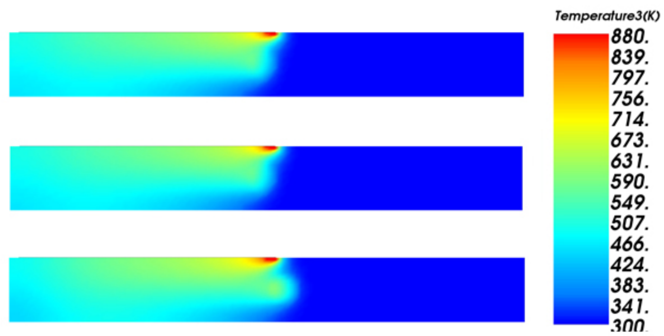

FIGURE 4: The transient temperature distribution is shown when the weld is at the mid-point of the top edge for three designs. The top image is with no side heaters. Red and yellow axes are X and Y. The middle image is with side heaters for Design A. The bottom image is for side heaters for Design B.

FIGURE 5: The effective or von Mises stress distribution is shown when the weld is at the end of the welding for Design A.

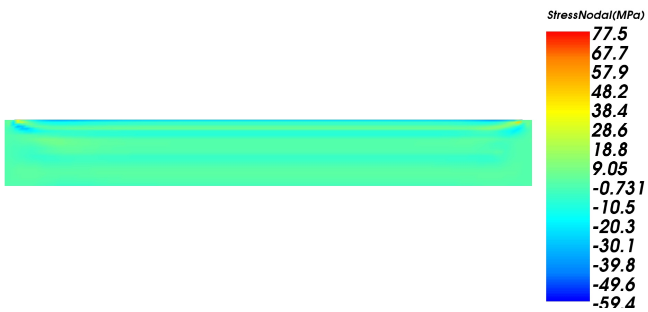

FIGURE 6: The longitudinal stress or σ_{xx} distribution is shown at the end of the welding for Design A.

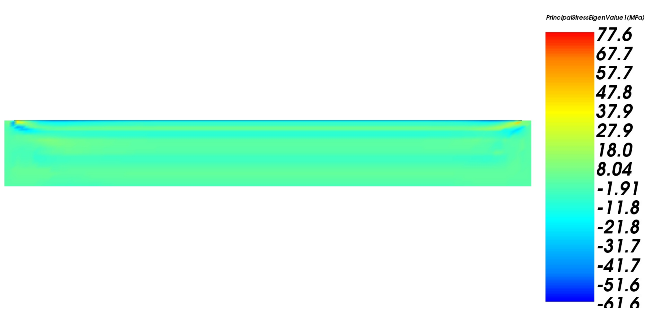

FIGURE 7: The maximum principal tensile stress distribution is shown at the end of the welding for Design A.

6.1 Optimization Project 17

In this project the response surface is computed for a complete 17×17 grid of design for the side heaters located at X = -0.0254 m, Y = 0.0762 m. The objective function, $abs(Y_{camb})$, absolute value of the of the Y-displacement along the bottom of the bar. The objective is to minimize this displacement, i.e., the camber caused by the weld. The response surface parameters

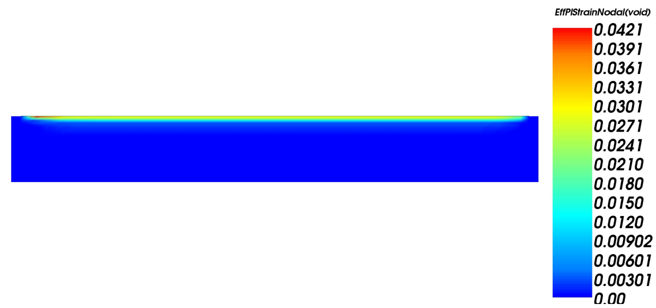

FIGURE 8: The effective plastic strain distribution is shown at the end of the welding for Design A.

radius and flux have the values listed in Table 1.

TABLE 1: Design variables setup for computing response surface, Project 17.

Variable	Init Value	Min Value	Max Value	Min Incr
Radius (m)	0.03	0.02	0.04	0.00125
Flux (w/m^2)	593,750.0	296,875.0	890,625.0	37,109.375

The computed objective function values are shown in Fig. 9. There are many radius-flux pairs that give final distortion near zero. The global minimum objective value 8.00676e-06 m is found at the grid cell (6 -1) which corresponds to the radius (R) = 0.0375 m and flux value (F) = 556,640.625 $W/(m^2)$.

6.2 Design of Optimal Experiment Given the Response Surface

Having computed a response surface for a given design space, how should one choose the set of physical experiments to run to validate the computer model? In other words, how should one choose the DOE matrix for the physical experiment to validate the computer model?

The answer requires an understanding of the various types of errors that could be involved. When one does the experiment, how accurately can the control variables be controlled. For example, what is the probable distribution of the welding current, voltage and weld speed when one when specifies values of say 260 A, 23 V, 7.1 mm/s? One might assume a Gaussian distribution for each variable defined by a mean value and a standard deviation.

There are errors in the numerical analysis. Reference [19] describes the validation of the computer model for computing the

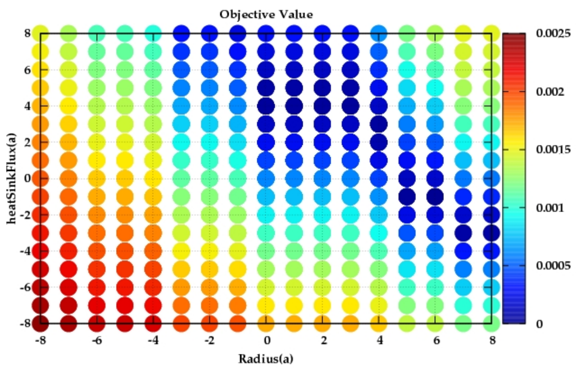

FIGURE 9: Objective values of camber computed in Project 17. A 2D design space has been discretized into a 17×17 grid. The objective function has been evaluated at all 289 points in the grid.

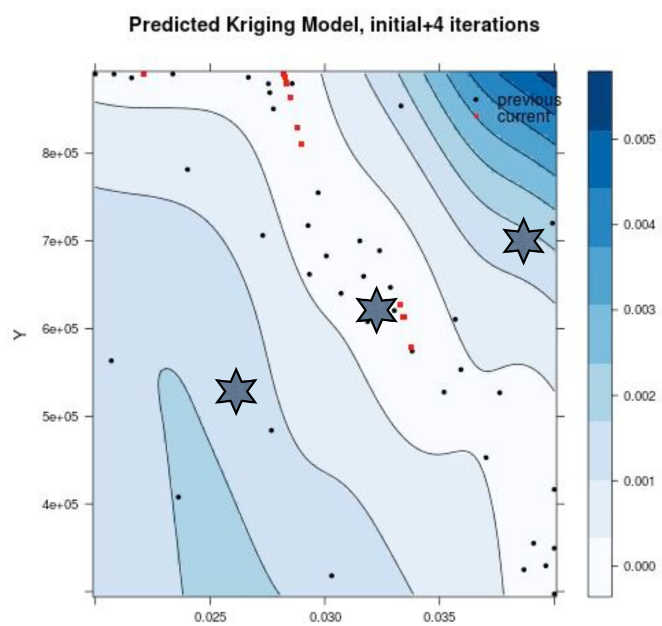

FIGURE 10: Design space for transverse displacement of side heat on horizontal axis and side heater thermal flux on the vertical axis. Each point is a distinct design of a side heater. The red points were computed by the kriging algorithm to minimize the uncertainty in the design space. The black points were evaluated in previous kriging iterations. The colours are values of the objective function for the camber. The three large stars are suggested designs for performing a physical test to validate the computer model.

camber in the edge weld of this bar with no side heater. Figure 1 shows that the model quite accurately predicted the transient mid-length deflection. This suggests that the errors in the model with no side heaters are relatively small.

However, the computer model in [19] does not consider errors in the weld parameters such as welding current, voltage and weld speed. Other errors could be expected, possibly in the geometry of the bar, the initial straightness of the bar. If one understood the most important probable errors, how should one choose the DOE matrix for the physical experiment?

The work of Taguchi [26] and Box [13] are the obvious starting point because these authors and others that have followed have devoted their lives to choosing a DOE matrix that provides the most information at the minimum cost. One of the first things to decide is how many experiments one is prepared to do, i.e., how much time and money can be devoted to the physical experiments? Of course the answer depends on the risk-reliability requirements of the problem. If the design is part of a satellite, in which the reliability requirements are very high, one would likely be prepared to pay more than if the design was for a farm tractor where the costs of a failure are much lower.

For discussion, let's assume that we choose Taguchi's $L_9(3^4)$ DOE matrix [26] for four design variables with nine tests or rows and tree levels for each design variable. For this choice, one must choose the 'centre point' and the two levels for each of the four design variables. Looking at the 2D sub-space of the 4D response surface shown in Fig. 10, one choice would be to pick the centre point with a minimum value of the objective function. Then choose two other points with values levels fairly far from the minimum. One would like to choose values for the levels that are large wrt to the uncertainty in the experimentally measured camber but not so large that the local behaviour of the response surface is not captured by a quadratic polynomial.

7 Summary

It has been shown that some but not all side heater designs that minimize distortion also minimize three measures of residual stress. The existence of a design that minimizes both distortion and residual stress suggests that an optimization to minimize both distortion and residual stress would generate a smaller set of optimal designs than an optimization to minimize only distortion. It is an open question if such an optimal design would be unique. In future work we plan to address address this question by running optimizations to minimize residual stress in welds.

It is not necessary for a side heater to generate plastic strain. It can be sufficient if the side heater changes the evolution of plastic strain near the weld pool by changing the stress state near the weld pool.

The designs with higher values of residual stress had larger values of effective plastic strain generated by the side heaters and possibly in the weld pool region. It is an open question if minimizing effective plastic strain minimizes residual stress. It is an open question if such non-optimal designs have more damage and thus are more susceptible to some failure mechanisms. In other words, could minimization of residual stress in welds also minimize the risk of failure by certain failure mechanisms.

This optimization methodology can be applied immediately to optimize any objective function that can be computed for any design in a design space defined by design variables that are parameters in VrWeld.

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